The general form for a confidence interval is

**Point Estimate ± Margin of Error**

**Point Estimate ± (Critical Value)⋅(Standard Deviation of the Statistic)**

± ×

The critical value is the number *z*-value needed to “capture” the middle C% of the distribution.

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| When constructing a 90% confidence interval using the *z*-distribution, use  *z*\* = *z*0.05 = 1.645. |  |
| When constructing a 95% confidence interval using the *z*-distribution, use  *z*\* = *z*0.025 = 1.960. |  |
| When constructing a 99% confidence interval using the *z*-distribution, use  z\* = *z*0.005 = 2.576. |  |

When documenting the solution to any inference procedure, use the C-C-C-C method:

* **Choose** the correct inference procedure.
* **Check** the conditions for the inference procedure.
* **Carry** out the inference procedure.
* State your **conclusion** by interpreting the confidence interval in context.

**A *z*-Confidence Interval for a Population Mean: σ known or assumed**

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| **A *z*-Confidence Interval for a Population Mean: σ known or assumed**  When the conditions are met and the population standard deviation is known or assumed, a *C*% confidence interval for the unknown mean μ is ± × where *z*\* is the critical value for the *z*-distribution, with C% of the area between –*z*\* and *z*\*. We call the quantity × the **margin of error**. The conditions for this inference procedure are   1. A random sample has been selected from the population. 2. The 10% condition: Check that *N* > 10*n*. 3. The population has an approximate Normal distribution OR the sample size is large (*n* ≥ 30). |

**Example 1**

Consider a production of 10,000 batteries. We want to estimate the true mean lifetime of all 10,000 batteries from a sample of size *n* = 9. The following table outlines our sample data:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Battery #** | 1922 | 3950 | 3405 | 7562 | 8713 | 9640 | 9125 | 3142 | 5448 |
| **Lifetime (min)** | 1017 | 973 | 1102 | 979 | 935 | 1032 | 1036 | 978 | 1082 |

The sample mean lifetime is 1014.89 min. We will use the population standard deviation of σ = 90 minutes to construct a 90% confidence interval for the true mean lifetime of all 10,000 batteries.

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| **Solution**  **Choose the correct inference procedure.**  We will calculate a 90% confidence interval for a population mean with a known value of σ.  **Check the conditions**   * A simple random sample was selected. ✓ * 10% condition: There are indeed more than 10*n* = 90 batteries in the population. ✓ * Though the sample size is not large, we do know that the population has an approximate Normal distribution.   **Carry out the inference procedure**  For a 90% *z*-confidence interval, use *z*\* = 1.645.  ± ×  **State the conclusion by interpreting the confidence interval**  We are 90% confident that the interval, 965.5 minutes to 1064.2 minutes, contains the true mean lifetime of all 10,000 batteries. |

**Example 2**

In the calculation of the confidence interval in example 1, identify the value of each of the following.

1. The point estimate of the parameter. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. The critical value. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. The standard deviation of the population. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. The margin of error. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 3**

In example 1, we interpreted the 90% confidence interval. Now interpret the confidence level of 90%.

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| **Answer**  The method used to produce the interval will contain the true mean lifetime of all 10,000 batteries in 90% of all possible samples of size *n* = 9 from this population. |

To illustrate the meaning of a confidence level, here are the confidence intervals based on 20 different simple random samples of size *n* = 9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Sample #** | **Point Estimate** | **Margin of Error** | **Lower CL** | **Upper CL** |  |
| 1 | 1014.89 | 49.35 | 965.5 | 1064.2 |  |
| 2 | 992.89 | 49.35 | 943.5 | 1042.2 |  |
| 3 | 1014.67 | 49.35 | 965.3 | 1064.0 |  |
| 4 | 1036.89 | 49.35 | 987.5 | 1086.2 |  |
| 5 | 1015.78 | 49.35 | 966.4 | 1065.1 |  |
| 6 | 953.00 | 49.35 | 903.7 | 1002.4 |  |
| 7 | 1002.22 | 49.35 | 952.9 | 1051.6 |  |
| 8 | 975.78 | 49.35 | 926.4 | 1025.1 |  |
| 9 | 944.11 | 49.35 | 894.8 | 993.5 |  |
| 10 | 1022.44 | 49.35 | 973.1 | 1071.8 |  |
| 11 | 998.44 | 49.35 | 949.1 | 1047.8 |  |
| 12 | 987.56 | 49.35 | 938.2 | 1036.9 |  |
| 13 | 999.00 | 49.35 | 949.7 | 1048.4 |  |
| 14 | 1013.33 | 49.35 | 964.0 | 1062.7 |  |
| 15 | 1040.00 | 49.35 | 990.7 | 1089.4 | ← Hmmmmm!!!! |
| 16 | 1009.78 | 49.35 | 960.4 | 1059.1 |  |
| 17 | 963.67 | 49.35 | 914.3 | 1013.0 |  |
| 18 | 1030.22 | 49.35 | 980.9 | 1079.6 |  |
| 19 | 970.89 | 49.35 | 921.5 | 1020.2 |  |
| 20 | 1020.00 | 49.35 | 970.7 | 1069.4 |  |

Examine the 20 confidence intervals.

How many of these confidence intervals contain the true mean lifetime of μ = 990 minutes? \_\_\_\_\_\_\_

What percent of these confidence intervals contain the true mean lifetime of μ = 990 minutes? \_\_\_\_\_

**Example 4**

**Interpreting the Confidence Interval: Mark each statement as CORRECT (C) or INCORRECT (I)**

\_\_\_ After taking many SRS of size 9 from the population of interest, 95% of the constructed intervals can be expected to have captured the true mean lifetime of all batteries.

\_\_\_ I am 95% confident that interval 965.5 minutes to 1064.2 minutes contains the true mean lifetime.

\_\_\_ The confidence interval was calculated using a method that will capture the true population parameter in 95% of all possible samples.

\_\_\_ 95% of all batteries have a lifetime between 965.5 minutes and 1064.2 minutes.

\_\_\_ The probability that the true mean lifetime falls between 965.5 minutes and 1064.2 minutes is 0.95.

\_\_\_ There is a 95% chance that the true mean lifetime is between 965.5 minutes and 1064.2 minutes

\_\_\_ 95% of the time the true mean lifetime of all batteries is between 965.5 and 1064.2 minutes.

**Example 5: The Margin of Error**

Recall that the margin of error is given by MOE = × .

1. Refer to example 1. Using *n* = 9 and σ = 90 minutes, compute the margin of error for both a 95% confidence interval and a 99% confidence interval. Enter your answers in the table below. What do you notice?
2. Using a confidence level of 90% and σ = 90 minutes, compute the margin of error for samples of size *n* = 25 and *n* = 81. What do you notice?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Margin of error for**  ***n* = 9 and σ = 90 minutes** | |  | **Margin of error for**  ***C* = 90% and σ = 90 minutes** | |
| **Confidence Level** | **Margin of Error** | **Sample Size** | **Margin of Error** |
| 90% | 49.35 minutes | *n* = 9 | 49.35 minutes |
| 95% |  | *n* = 25 |  |
| 99% |  | *n* = 81 |  |

For a fixed sample size and fixed standard deviation, as the confidence level increases, the margin of error \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (increases/decreases).

For a fixed confidence level and fixed standard deviation, as the sample size increases, the margin of error \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (increases/decreases).

**Example 6**

Using a confidence level of 90% and σ = 90 minutes, we computed the margin of error for samples of size *n* = 25 and *n* = 81 and noticed that as the sample size increases, the margin of error decreases.

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| --- | --- |
| **Sample Size** | **Margin of Error** |
| *n* = 9 | = 49.35 minutes |
| *n* = 25 | = 29.61 minutes |
| *n* = 81 | = 16.45 minutes |

From MOE= × , we see that the margin of error is inversely proportional to the sample size.

* If you want to cut the margin of error in half, you must increase the sample size by a factor of 4.
* If you want to reduce the margin of error by a factor or 3, you must increase the sample size by a factor of 9.

Suppose that we wanted to determine the minimum sample size required to estimate the true mean lifetime of all the batteries to within a specified margin of error *M* with a *C*% level of confidence. Thus, we want to find *n* so that

M= × .

Multiply both sides of the equation by .  ×

Divide both sides of the equation by *M*. 

Finally, solve for *n* by squaring both sides of the equation. 

Use the equation to find the sample size needed to estimate the population mean to within a specified margin of error *M* with a confidence level of *C*%. This formula will rarely yield a whole number. Since a sample size must be a whole number, always round your answer **up** to the nearest whole number so that the margin of error is less strict than *M*.

**Example 7: A Sample Size Calculation**

Refer to our population of 10,000 batteries. Find the minimum sample size needed to estimate the true mean lifetime to within 15 minutes of actual value with a 95% level of confidence.

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| **Solution**  We will use the formula where σ = 90 minutes, *M* = 15 minutes,  and *z*\* = 1.960 (the critical value of a 95% confidence interval).  As expected, the result is not a whole number. Round this number up to 139 batteries.  **Answer**  A minimum sample of size 139 batteries is needed to estimate the true mean lifetime to within 15 minutes of the actual value with a confidence level of 95%. |